

Multiple light scattering in laser particle sizing

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Multiple light scattering is an important issue in modern laser diffraction spectrometry. Most laser particle sizers do not account for multiple light scattering in a disperse medium under investigation. This causes an underestimation of the particle sizes in the case of high concentrations of scatterers. The retrieval accuracy is improved if the measured data are processed with multiple-scattering algorithms that treat multiple light scattering in a disperse medium. We evaluate the influence of multiple light scattering on light transmitted by scattering layers. The relationships among different theories to account for multiple light scattering in laser particle sizing are considered. © 2001 Optical Society of America

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1. Introduction

Small-angle laser diffraction is often used for determining particle size distributions (PSD's) in disperse media.¹ The retrieval schemes in this case are based on the strong dependence of Fraunhofer diffraction patterns of large particles on their sizes. Indeed it follows for the scattered light intensity in the case of monodisperse ensembles of large spheres² that

$$I(\theta, a) = N \frac{J_1^2(\alpha\theta)a^2}{\theta^2 R^2} I_0 V, \quad (1)$$

where J_1 is the Bessel function; a is the radius of particles; $\alpha = ka$, where $k = 2\pi/\lambda$, where λ is the wavelength of incident light; θ is the scattering angle, I_0 is the intensity of the incident light, N is the number of particles per unit volume, V is the volume of a scattering medium, and R is the distance to the observation point. The intensity $I(\theta, a)$ is proportional to the square of the Bessel function and therefore has several minima and maxima. The positions of minima and maxima depend on size parameter α . The

first minimum occurs at scattering angle $\theta = A/\alpha$, where $A \approx 3.832$.

We can account for the polydispersity of a disperse medium by averaging the scattered light intensity [see Eq. (1)] with the PSD $q_0(a)$:

$$\bar{I}(\theta, a) = \int_0^\infty I(\theta, a) q_0(a) da. \quad (2)$$

Equation (1) is valid under the following assumptions:

$$\alpha \gg 1, 2\alpha|m - 1| \gg 1, \quad (3)$$

$$\frac{l}{L} \gg 1, \quad (4)$$

$$\frac{D}{d} \gg 1, \quad (5)$$

where $m = n - ik$ is the refractive index of the particles, L is the geometric thickness of a scattering layer, l is the mean photon free path length, d is the diameter of the particles, and D is the distance between particles. Note that, owing to assumption (3), the Fraunhofer diffraction pattern of a particle coincides with the Fraunhofer diffraction pattern for a black screen with radius a . The task of the retrieval algorithm of the laser diffraction spectrometry is to find the radius (or radii distributions) of such screens. Equations (4) and (5) state, respectively, that the multiple scattering³ and correlation effects^{4,5} are negligible.

In many problems, such as those encountered in particle sizing for Diesel injectors and heavy fuel oil

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atomizers, the correlation effects can be neglected ($D \gg d$), but condition (4) is often not fulfilled. Thus one needs to account for multiple light scattering and obtain a more general formula than the simple single-scattering approximation presented by Eq. (1).

This could be done by different means. Interesting approaches to dealing with the problem have been proposed by Felton *et al.*⁶ and Hirleman.^{7,8} Optical particle-sizing techniques based on the use of the radiative-transfer equation³ have been studied by Belov *et al.*,⁹ Vagin and Veretennikov,¹⁰ and Schnablegger and Glatter.¹¹

We show that analytical solutions^{7–11} are identical at small angles and can be mutually derived from one another. The simplified equation for the calculation of small-angle light-scattering patterns of multiple light-scattering media with large spherical particles is derived.

2. Multiple Light Scattering

A. Different Forms of Solutions

The radiative-transfer equation (RTE) for a plane-parallel light-scattering layer can be written in the following form³:

$$\cos \vartheta \frac{dI(\tau, \vartheta)}{d\tau} = -I(\tau, \vartheta) + \frac{\omega_0}{2} \int_0^\pi I(\tau, \vartheta') p(\theta) \sin \vartheta' d\vartheta', \quad (6)$$

where ϑ is the observation angle; $\omega_0 = \sigma_{\text{sca}}/\sigma_{\text{ext}}$ is the single-scattering albedo, where σ_{sca} is the scattering coefficient and σ_{ext} is the extinction coefficient; $p(\theta)$ is the azimuthally averaged phase function; $\tau = \sigma_{\text{ext}} L$ is the optical thickness, where L is the geometric thickness of a layer; and I is the light intensity. The scattering angle θ is related to angles ϑ, ϑ' by the following simple equation³:

$$\theta = \arccos[\cos \vartheta \cos \vartheta' + \sin \vartheta \sin \vartheta' \cos(\vartheta - \vartheta')] \quad (7)$$

where ϑ and ϑ' are azimuths.

The specific form of the RTE presented by Eq. (6) is valid only at the normal illumination of a light-scattering plane-parallel layer by a plane wave. The transmitted diffuse light intensity does not depend on the azimuth in this case owing to the symmetry of the problem.

We consider the angular dependence of function $I(\tau, \vartheta)$ at small values of the observation angle ϑ . Thus it is assumed that

$$\cos \vartheta \frac{dI(\tau, \vartheta)}{d\tau} \approx \frac{dI(\tau, \vartheta)}{d\tau}, \quad (8)$$

where we accounted for the approximate equality: $\cos \vartheta \approx 1$, which is valid as $\vartheta \rightarrow 0$. This approximation is called the small-angle approximation³ of the radiative-transfer theory. Equation (6) under

approximation (8) can be solved analytically. The answer is well known (see, e.g., Refs. 3 and 12):

$$I(\tau, \vartheta) = \frac{I_0}{2\pi} \sum_{j=0}^{\infty} \Xi_j(\tau) P_j(\cos \vartheta), \quad (9)$$

where

$$\Xi_j(\tau) = \left(j + \frac{1}{2}\right) \exp[-(1 - \omega_0 p_j)\tau], \quad (10)$$

$$p_j = \frac{1}{2} \int_0^\pi p(\theta) P_j(\cos \theta) \sin \theta d\theta. \quad (11)$$

Here $P_j(\cos \theta)$ is the Legendre polynomial.

Our prime interest here is the incoherent intensity of multiply scattered light. Thus we remove the coherent component³ (or direct light),

$$I_0 \exp(-\tau) \delta(\cos \vartheta - 1) = \frac{I_0}{2\pi} \exp(-\tau) \times \sum_{j=0}^{\infty} \left(j + \frac{1}{2}\right) P_j(\cos \vartheta), \quad (12)$$

from Eq. (9) and obtain

$$I'(\tau, \vartheta) = \frac{I_0}{2\pi} \sum_{j=0}^{\infty} \Xi_j'(\tau) P_j(\cos \vartheta), \quad (13)$$

where

$$\Xi_j'(\tau) = \left(j + \frac{1}{2}\right) e^{-\tau} [\exp(\omega_0 p_j \tau) - 1]. \quad (14)$$

Equations (13) and (14) were obtained by Hartel¹² and used in Ref. 11 for solution of the inverse problem. Note that numerical calculations of coefficients p_j [see Eq. (11)] are complicated in the case of large particles because the phase function is strongly peaked in the forward direction ($\theta = 0$) at $a \gg \lambda$.

Let us derive the integral form of Eq. (13). It follows as $\theta \rightarrow 0$:

$$P_j(\cos \theta) \rightarrow J_0 \left[\theta \left(j + \frac{1}{2}\right) \right]. \quad (15)$$

Thus we can introduce the following function, which follows from Eqs. (11) and (15):

$$g(\sigma) = \frac{1}{2} \int_0^\pi p(\theta) J_0(\sigma \theta) \theta d\theta, \quad (16)$$

where $\sigma = j + 1/2$ and we extend the upper limit of the integration in Eq. (11) to infinity. This is possible owing to a sharp peak in the phase function $p(\theta)$ [see Eq. (1)] at scattering angle $\theta \approx 0$. Also we use the continuous function $g(\sigma)$ instead of discrete numbers

p_j [see Eq. (11)]. It follows from Eq. (13) and the Euler sum formula

$$\sum_{l=0}^{\infty} f\left(l + \frac{1}{2}\right) \approx \int_0^{\infty} f(p) dp$$

that

$$I'(\tau, \vartheta) = \frac{I_0}{2\pi} e^{-\tau} \int_0^{\infty} \{\exp[\tau \omega_0 g(\sigma)] - 1\} J_0(\sigma \vartheta) \sigma d\sigma. \quad (17)$$

Equation (17) is more convenient for applications than Eq. (13). Note that the simple form of Eq. (1) allows for the analytical integration in Eq. (16) (see Subsection 3.A).

Equation (17) was used in Refs. 9 and 10 for solution of the inverse problem taking into account multiple light scattering. Thus it is obvious that approaches to optical particle sizing in Refs. 9, 10, and 11 are based on asymptotically equivalent formulas [see Eqs. (13) and (17)]. Note that Eq. (17) can be obtained directly from Eq. (6) by the use of Fourier transform techniques.¹³

Let us now consider the expansion of the exponent in Eq. (17):

$$\exp[\tau \omega_0 g(\sigma)] = \sum_{n=0}^{\infty} \frac{\tau^n \omega_0^n g^n(\sigma)}{n!}. \quad (18)$$

It follows from Eqs. (17) and (18) that

$$I'(\tau, \vartheta) = \frac{I_0}{2\pi} e^{-\tau} \sum_{n=1}^{\infty} \frac{\tau^n \omega_0^n}{n!} H_n(\vartheta), \quad (19)$$

where

$$H_n(\vartheta) = \int_0^{\infty} g^n(\sigma) J_0(\sigma \vartheta) \sigma d\sigma. \quad (20)$$

Note that the successive scattering order expansion [Eq. (19)] coincides¹⁴ with Hirleman's solution [see Eq. (33) in Ref. 7] and provides a new way to calculate the function $H_n(\vartheta)$.

B. Single-Scattering Approximation

It follows from Eq. (19) at $n = 1$ and small values of τ (the single-scattering approximation) that

$$I'(\tau, \vartheta) = \frac{I_0}{2\pi} \tau \omega_0 \int_0^{\infty} g(\sigma) J_0(\sigma \vartheta) \sigma d\sigma, \quad (21)$$

or using the equality $\tau \omega_0 = \sigma_{\text{sca}} L$ yields

$$I'(\tau, \vartheta) = \frac{I_0}{2\pi} \sigma_{\text{sca}} L p(\vartheta), \quad (22)$$

where [see Eq. (16)]

$$p(\vartheta) = \int_0^{\infty} g(\sigma) J_0(\sigma \vartheta) \sigma d\sigma \quad (23)$$

is the phase function. On the other hand, it is well known² that the phase function of the Fraunhofer diffraction process has the following form:

$$p(\vartheta) = \frac{4J_1^2(\vartheta \rho)}{\vartheta^2}. \quad (24)$$

Thus for the intensity within the framework of the single-scattering approximation one can obtain

$$I(\tau, \vartheta) = p(\vartheta) a^2 N L I_0, \quad (25)$$

where we use the equality $\sigma_{\text{sca}} = 2\pi N a^2$, which is valid at $a \gg \lambda$.³

It follows that Eq. (25) [see also Eq. (24)] coincides with Eq. (1) at $L = V/(4R^2)$. Different multipliers $[L$ and $V/(4R^2)]$ are due to the different symmetry of the problem associated with Eqs. (1) and (6). Note that normalized intensities $i(\tau, \vartheta) = I(\tau, \vartheta)/I(\tau, 0)$ defined by Eqs. (1) and (25) coincide.

Thus one can see that Eqs. (13), (17), and (19) are special cases of the solution of the RTE [Eq. (6)]. They have already been used for development of different inversion schemes within the framework of laser diffraction spectrometry.^{8,9,11}

3. Multiple Fraunhofer Diffraction

A. Monodisperse Media

The main task in this section is to study Eq. (17) in more detail for the multiply scattered light intensity $I(\tau, \vartheta)$. This formula can be applied to many kinds of disperse media with different types of forward peaked phase function, including Rayleigh–Gans phase functions.³

Let us now consider a special case of Eq. (17) with the phase function $p(\vartheta)$, defined in Eq. (24). It follows for the Fourier–Bessel transform of the phase function (see Fig. 1) that

$$g(z) = \begin{cases} \frac{2}{\pi} [\arccos(z) - z(1 - z^2)^{1/2}] & z \leq 1 \\ 0 & z > 1 \end{cases}, \quad (26)$$

where $z = \sigma/\eta$ and $\eta = 2\alpha$. The single-scattering albedo ω_0 is equal to 0.5 within the framework of the Fraunhofer approximation. Thus it follows from Eq. (17) for the intensity of the transmitted diffused light that

$$I'(\tau, \vartheta) = \frac{I_0}{2\pi} e^{-\tau} \int_0^{\infty} \left\{ \exp\left[\frac{\tau g(\sigma/\eta)}{2}\right] - 1 \right\} J_0(\sigma \vartheta) \sigma d\sigma \quad (27)$$

or

$$I'(\tau, \vartheta) = C e^{-\tau} \int_0^1 \left[\exp\left[\frac{\tau g(z)}{2}\right] - 1 \right] J_0(bz) z dz, \quad (28)$$

where $b = 2ka\vartheta$, $C = [(2\alpha^2)/\pi] I_0$, and the function $g(z)$ is presented by Eq. (26) and Fig. 1. One can observe the results of calculations of the normalized intensity $i(\tau, \vartheta) = I(\tau, \vartheta)/I(\tau, 0)$ obtained with Eq. (28)

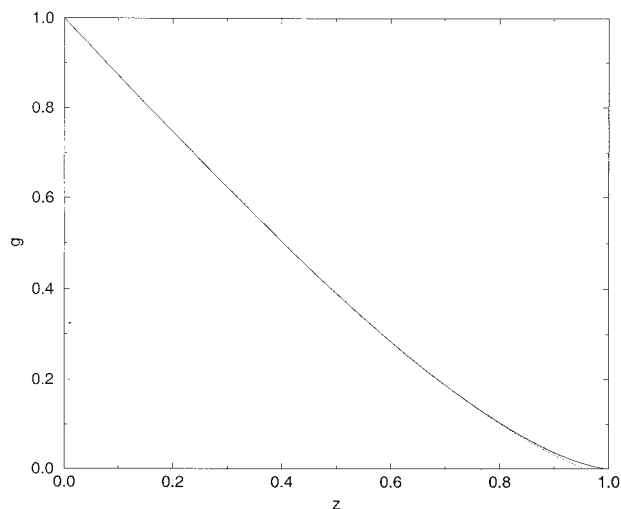


Fig. 1. Fourier-Bessel transform of the Fraunhofer phase function [Eq. (24)] obtained with the exact solution [Eq. (26)] and the approximate formula [Eq. (34)].

for different values of the optical thickness τ as a function of parameter b in Fig. 2. It follows that multiple light scattering causes broadening of the angular intensity distributions. The same effect could be due to the decrease in particle size [see Eq. (1)]. Thus multiple-scattering broadening is responsible for the underestimation of the particle size by laser diffraction spectrometers, designed for thin scattering layers, if they are used for large values of the optical thickness.

Let us define the dimensionless halfwidth parameter h by

$$i(b = h) = 0.5. \quad (29)$$

It can be calculated from Eq. (28) for different τ . The value of h increases with the optical thickness (see Fig. 3 and Table 1). Note that Table 1 can be used to estimate the radii of monodisperse particles from

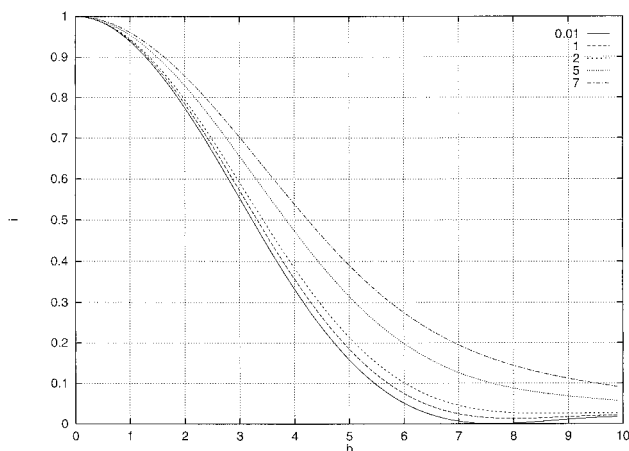


Fig. 2. Dependence of the normalized intensity on parameter $b = 2ka\vartheta$ obtained with Eq. (28) at different values of optical thickness $\tau = 0.01, 1, 2, 5, 7$.

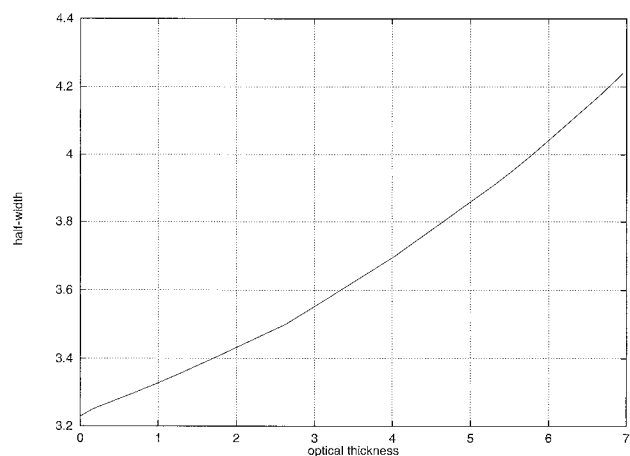


Fig. 3. Dependence of half-width h of the angular spectrum of the transmitted light on optical thickness τ .

measurements of optical thickness τ and observation angle ϑ_0 at which $i(\vartheta_0) = 0.5$:

$$a = \frac{h(\tau)}{4\pi\vartheta_0} \lambda. \quad (30)$$

The value of h in Eq. (30) should be selected from Table 1, depending on optical thickness. Also one can use $h = 3.23614 + 0.0768\tau + 0.00937\tau^2$, which was obtained by fitting the data in Table 1 (see Fig. 3). The same method can be applied for estimation of the mode radius of narrow particle-size distributions. However, note that the values in Table 1 ignore light rays reflected and transmitted by particles. They account only for the diffraction component of the total scattered-light intensity.

It is useful to estimate the upper limit of optical thickness τ^* , where multiple scattering can be neglected. It follows from Fig. 2 that the influence of multiple light scattering is larger for larger values of

Table 1. Dependence of the Half-Width Parameter h of the Angular Distribution of Transmitted Light on Optical Thickness τ for Disperse Media with Monodispersed Spheres

τ	h
0.01	3.23
0.5	3.28
1.0	3.32
1.5	3.38
2.0	3.43
2.5	3.49
3.0	3.55
3.5	3.62
4.0	3.69
4.5	3.77
5.0	3.85
5.5	3.94
6.0	4.03
6.5	4.14

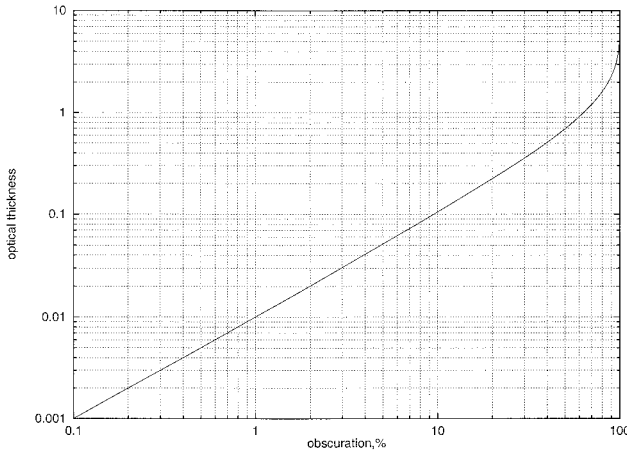


Fig. 4. Dependence of the optical thickness on obscuration.

parameter $b = 2ka\vartheta$. One can introduce the following criteria on the basis of data in Table 1:

$$1 - \frac{h(\tau^*)}{h(0.01)} \leq \varepsilon, \quad (31)$$

where the value of ε depends on the tolerable error for the retrieval scheme. Clearly, the value of h at $\tau = 0.01$ corresponds to the case of single scattering; e.g., if one allows for the difference in half-widths $\varepsilon = 1\%$, it follows that $\tau^* \leq 0.5$ to fulfill Eq. (31). Malvern Instruments advises that the upper limit of the obscuration $\mu = [1 - \exp(-\tau)] \times 100\%$ should be 50% to avoid multiple light scattering.¹⁵ The optical thickness τ^* is equal to 0.7 in this case (see Fig. 4) and $\varepsilon \approx 2\%$ [see inequality (31) and Table 1]. Bayvel *et al.*^{16,17} found that it is possible to obtain reliable results of the inversion even at $\mu = 90\%$ (or at $\tau^* = 2.3$, see Fig. 4). One can find that the value of ε is equal to 7% in this case.

In conclusion, Eq. (28) [see Eq. (26) as well] provides a simple and useful tool for studies of the contribution of multiple light scattering to Fraunhofer diffraction patterns of scattering media with monodisperse spheres.

B. Polydispersions

It is important to generalize Eq. (27) for the case of polydispersions. It could be done by averaging optical thickness τ and the Fourier–Bessel transform of the phase function $g(z)$ [see Eqs. (1) and (26)] with the PSD $q_0(a)$ (Ref. 20):

$$\langle \tau \rangle = \int_0^\infty \tau(a) q_0(a) da, \quad (32)$$

$$\langle g(\sigma) \rangle = \frac{\int_B^\infty g(\sigma/2ka) a^2 q_0(a) da}{\int_0^\infty a^2 q_0(a) da}, \quad (33)$$

where $B = \sigma/2k$ and $\tau(a) = 2\pi a^2 NL$ (Ref. 3) for large particles. One should substitute the values of Eqs. (32) and (33) [instead of τ and $g(\sigma/\eta)$] in Eq. (27) to obtain the expression for the intensity of the light transmitted through a polydisperse medium.

Thus it follows that the only complication in Eq. (27) for a polydispersed medium is that function $g(\sigma)$ is not described by the simple Eq. (26). One should use the ratio of integrals [Eq. (33)] in this more complex case [see Eq. (32)]. The optical thickness of a polydisperse medium is determined by the integral $\int_0^\infty a^2 q_0(a) da$. The integrals in Eqs. (32) and (33) can be evaluated analytically for some special cases.

For example, it could be done by expansion of the function $g(z)$ [see Eq. (26)] at small values of z :

$$g(z) = 1 + \sum_{j=0}^3 s_j z^{2j+1}, \quad (34)$$

where $s_0 = -(4/\pi)$, $s_1 = 2/(3\pi)$, $s_2 = 1/(10\pi)$, $s_3 = 1/(28\pi)$. Here we neglected terms of the order of z^9 and higher. The error in the replacement of Eq. (26) by Eq. (34) is small (see Fig. 1). The largest error is located at $z \approx 1$. However, the value of $g(z)$ is small at small values of $1 - z[g(1) = 0]$ [see Eq. (26)], and the correspondent contribution to Eq. (27) can be neglected.

It follows from Eqs. (33) and (34) that

$$\langle g(\sigma) \rangle = 1 + \sum_{j=0}^3 s_j \varphi(2j+1, B), \quad (35)$$

where

$$\varphi(n, B) = B^n \frac{\int_B^\infty a^{2-n} q_0(a) da}{\int_0^\infty a^2 q_0(a) da}. \quad (36)$$

The special function $\varphi(n, B)$ in Eq. (36) can be calculated analytically for many types of PSD. For example, it follows for the gamma PSD $q_0(a) = Aa^\mu \exp(-a\mu/a_0)$, where A is the normalization constant ($\int_0^\infty q_0(a) da = 1$), a_0 is the mode radius, and μ is the half-width parameter:

$$\langle g(\sigma) \rangle = 1 + \sum_{j=0}^3 s_j \varphi(2j+1, \beta),$$

$$\varphi(n, \beta) = \beta^n \frac{\int_\beta^\infty y^{2+\mu-n} \exp(-y) dy}{\int_0^\infty y^{2+\mu} \exp(-y) dy}, \quad (37)$$

where $\beta = (\lambda\mu\sigma)/(4\pi a_0)$. From Eq. (37) at $\mu > n - 3$ one can obtain

$$\varphi(n, \beta) = \beta^n \frac{\Gamma(\mu+3-n)[1 - P(\mu+3-n, \beta)]}{\Gamma(\mu+3)}, \quad (38)$$

where

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} \exp(-t) dt \quad (39)$$

is the gamma function, and

$$P(\alpha, x) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} \exp(-t) dt \quad (40)$$

is the incomplete gamma function. It follows at integer values of μ that

$$\Gamma(\mu) = (\mu - 1)!, \quad P(\mu, \beta) = 1 - \exp(-\beta) \sum_{l=0}^{\mu-1} \frac{\beta^l}{l!} \quad (41)$$

and [see Eq. (38)]

$$\varphi(n, \beta) = \beta^n \exp(-\beta) \frac{\Gamma(\mu + 3 - n)}{\Gamma(\mu + 3)} \sum_{l=0}^{\mu+2-n} \frac{\beta^l}{l!}. \quad (42)$$

Thus for the function $\langle g(\sigma) \rangle$ within the framework of the approximation being considered one can obtain

$$\begin{aligned} \langle g(\sigma) \rangle &= 1 + \sum_{j=0}^3 \sum_{l=0}^{\mu-2j+1} s_j \beta^{2j+l+1} \\ &\times \exp(-\beta) \frac{\Gamma(\mu - 2j + 2)}{\Gamma(\mu + 3)\Gamma(l + 1)}. \end{aligned} \quad (43)$$

Note that optical thickness $\langle \tau \rangle$ [Eq. (32)] is related to the volumetric concentration of particles c by a simple formula³:

$$\langle \tau \rangle = \frac{1.5cL}{a_{\text{eff}}}, \quad (44)$$

where

$$a_{\text{eff}} = \frac{\int_0^\infty a^3 q_0(a) da}{\int_0^\infty a^2 q_0(a) da} \quad (45)$$

is the effective (or Sauter) radius and c is the fraction of a unit volume, occupied by particles. It follows from Eq. (45) for the gamma PSD that

$$a_{\text{eff}} = a_0 \left(1 + \frac{3}{\mu} \right). \quad (46)$$

Equation (44) can be used for estimation of the concentration of particles c from measured values of L , τ , a_{eff} .

Thus it follows from Eqs. (27) and (44) [see Eq. (43)]

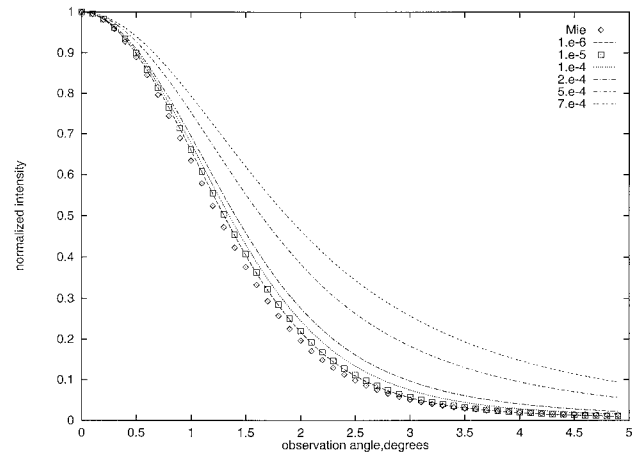


Fig. 5. Dependence of the normalized intensity on the observation angle obtained with the Mie theory (Mie) and Eqs. (47) and (43) for the gamma PSD $f(a) = Aa^6 \exp(-1.5a)$ at wavelength $\lambda = 0.628 \mu\text{m}$, the geometric thickness of a scattering layer $L = 4 \text{ cm}$, and different volumetric concentrations of particles $c = 0.000001$, $c = 0.00001$, $c = 0.0001$, $c = 0.0002$, $c = 0.0005$, $c = 0.0007$.

as well] for the intensity of the diffused transmitted light in the small-angle range that

$$\begin{aligned} I'(\vartheta) &= \frac{I_0}{2\pi} \exp\left(-\frac{3cL}{2a_{\text{eff}}}\right) \int_0^\infty \left\{ \exp\left[\frac{3cL\langle g(\sigma) \rangle}{4}\right] - 1 \right\} \\ &\times J_0(\sigma\vartheta) \sigma d\sigma. \end{aligned} \quad (47)$$

Results of calculations of normalized intensities $i(\vartheta) = [I(\vartheta, \tau)]/[I(0, \tau)]$ with Eq. (47) for polydispersions with the PSD $q_0(a) = Aa^6 \exp(-1.5a)$ are presented in Fig. 5 for different volumetric concentrations of scatterers c and a wavelength of $0.628 \mu\text{m}$. The geometric thickness of a layer and the effective radius of particles were equal to 4 cm and $6 \mu\text{m}$ [see Eq. (46)], respectively.

It follows from Fig. 5 that the increase in concentration of particles causes a broadening of the angular distribution. This is the same effect as for monodisperse media. We also present data for the normalized intensity calculated with the Mie theory for the same PSD, $\lambda = 0.628 \mu\text{m}$ and $n = 1.33$ (water drops) in Fig. 5. The small difference between the Mie curve and our curves at $c = 10^{-6}$, 10^{-5} ($\tau = 0.01$, 0.1) is related to the error of approximations in Eqs. (1) and (43).

5. Conclusion

The widely accepted approach to optical particle sizing is based on the single-scattering approximation. However, many natural media (e.g., atmospheric aerosols and clouds) are optically thick, and one should account for the multiple light scattering events in the retrieval schemes. This is important for laboratory measurements as well, because some disperse media (e.g., in the solid state) cannot be diluted.

We have found that different optical particle-sizing techniques⁷⁻¹¹ have the same ground, namely, the

small-angle approximate solution [Eq. (17)] of the radiative-transfer Eq. (7). Equation (17) can be applied for the solution of both direct and inverse problems of disperse media optics in multiple-scattering conditions. This formula was transformed to Eq. (47) for the simplification of calculations of the small angle transmitted intensity in the case of light-scattering media with polydisperse large spherical particles.

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